Space charge effects in 6D coolimg

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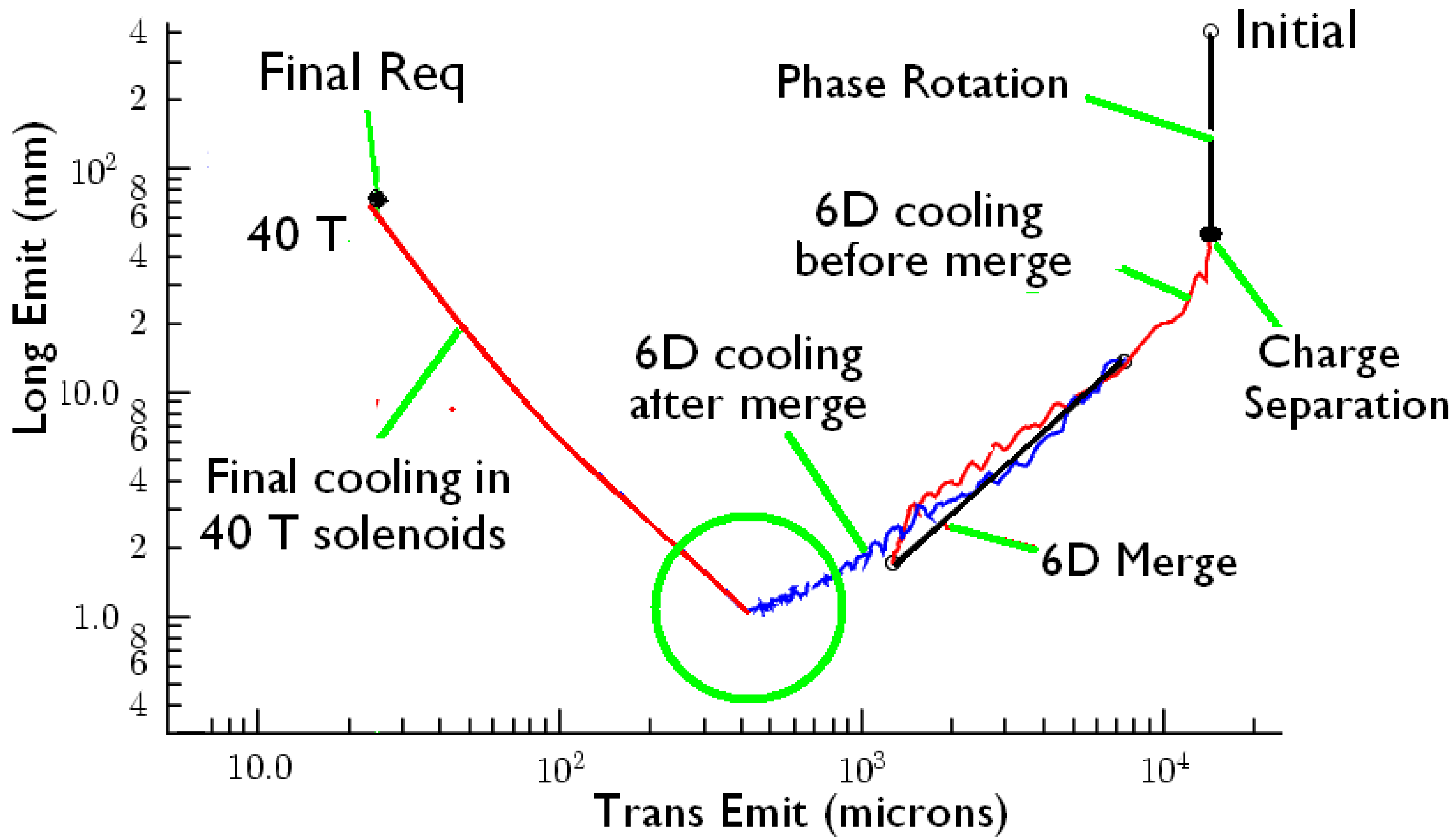
MAP Winter Meeting
SLAC



This is a plot from last year's Winter Collaboration Meeting in JLAB

Outline

- Introduction
- Transverse Space-Charge
- Longitudinal Space-Charge
- The Challenge: We should avoid cooling to low longitudinal emittance
- Solution
    1. New taper after merge with less emittance exchange
    2. Added new cooling lattice for lower transverse emittance
- Improvements if HTS used in 6D
- Conclusion

1) Introduction

- For $10^{34}\ s^{-1} cm^{-2}$ at 1.5 TeV, we need 2 $\times\ 10^{12}$ muons/bunch
- At end of 'Final Cooling' the bunch is long (4 m) giving a relatively moderate current and space charge effects
- But in late 6D cooling, the rms bunch length is only $\sigma_{ct} \approx 2$ cm
- And when losses in 'Final Cooling' and Acceleration are included the bunches have 4.87 10^{12} muons, giving a peak current of

$$\frac{4.87\ 10^{12}\ 1.6\ 10^{-19}\ \ 3\ 10^{8}}{\sqrt{2\pi}\ \ 0.02} = 4.6\ \mathrm{kA} \qquad \text{at only 160 MeV/c}$$

- This is a large current, at a low mom, so coherent effects can be expected

Transverse Space Charge Tune Shift

For a Gaussian longitudinal charge distribution with rms length of σ_z the tune shift per unit length is

$$\frac{d\nu_{\rm space}}{dL} = \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right) \frac{r_\mu}{4\pi\ \epsilon_\perp\ \beta_v \gamma^2}$$

For a cell of a periodic system, $\nu = \Delta\phi / L_{cell}$ the fractional tune shift is thus

$$\frac{\Delta\nu_{\rm space}}{\nu_{cell}} = \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right) \frac{r_\mu\ L_{cell}}{\phi_{cell}\ 4\pi\ \epsilon_\perp\ \beta_v \gamma^2}$$

Transverse tune shift in late 6D cooling

The phase advance per cell, as a function of momentum, in an RFOFO Guggenheim lattice ia from π to 2π. So the average phase advance per cell is 1.5 π and $\Delta\nu = \pm\pi/2$

$L_{cell} = 0.6875$ m long,
momentum= 207 MeV/c,
N_μ= 4.87 10^{12},
σ_{ct}= 1.87 cm, and
$\epsilon_\perp$= 400 μm

Giving $\Delta\nu/\nu = $ 0.068

a small fraction of the acceptance of $\pm$ 0.25/0.75 $=$ 0.33
So this is not a problem

Longitudinal Space Charge

Published calculations of longitudinal space charge are mostly given for uniform charge densities within 3 dimensional ellipsoids whose projections in all coordinates are parabolic:

$$\frac{dQ}{dx} = \rho_o \left(1 - \frac{z^2}{z_m^2}\right)$$

For these distributions the rate of change of the longitudinal space charge field, i.e the longitudinal defocusing force is

$$\mathcal{E}'_{sc} = \frac{d\mathcal{E}'_{sc}}{dz_{\rm lab}} = \left(\frac{3\ Q\ g}{8\ \pi\ \epsilon_o\ \gamma^2\ (z_m)^3}\right)$$

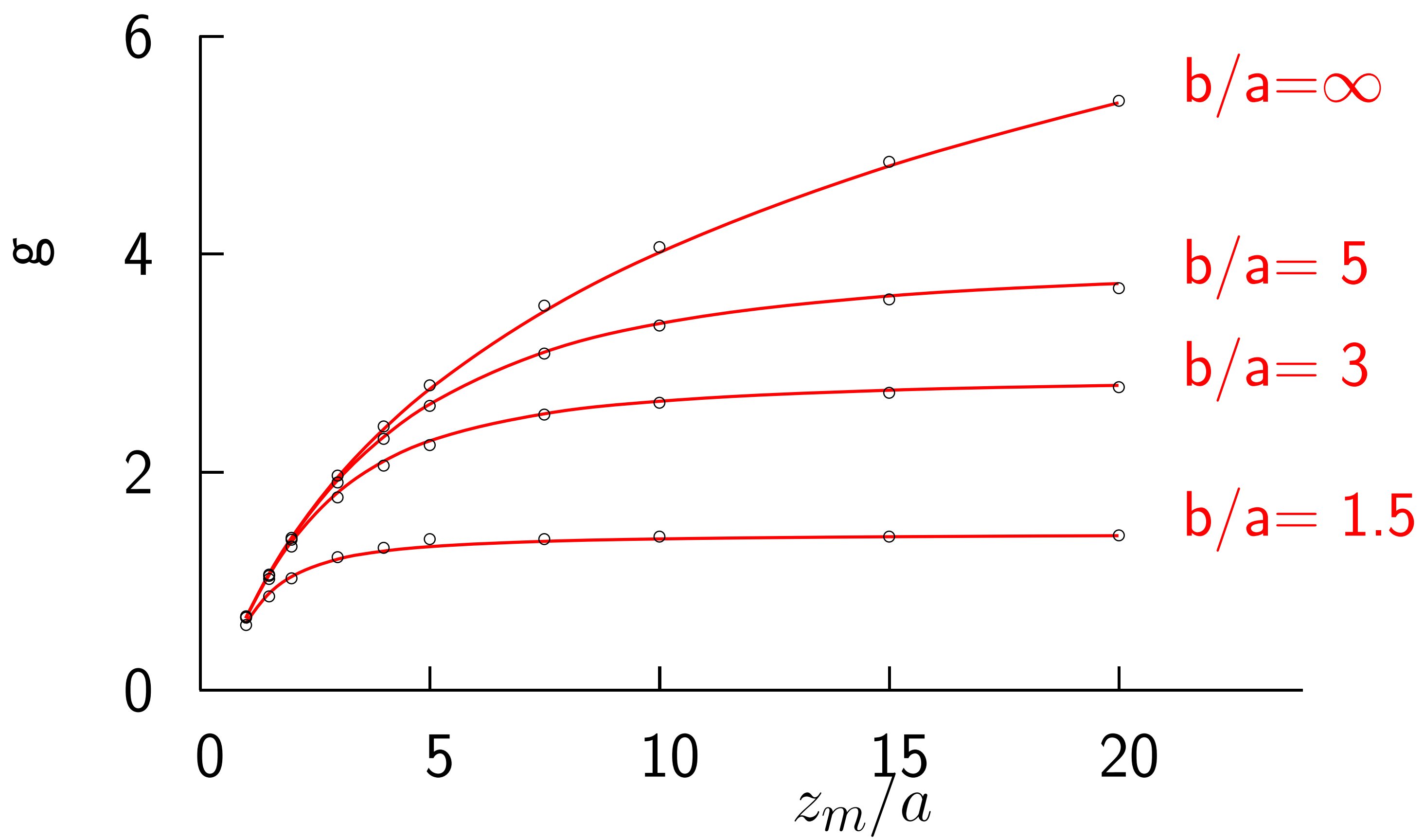
where g is a form factor that depends on z_m/a and b/a where z_m is the longitudinal, and a is the transverse size of the ellipsoid, and b is the assumed pipe radius

Note inverse cubic dependence on bunch length

Form factor g

In the absence of a pipe, the space charge, as given above, is linear in z with a fixed slope $\mathcal{E}'$ so that g is a constant for any given z_m/a

In a pipe it deviates from linearity, but average values of g have been numerically calculated (Allen) and shown below with a fit



rf Requirement

To reach the same performance as simulated without space charge, the needed ${\mathcal{E}'}_{\rm rf}$ must equal the simulated contribution ${\mathcal{E}'}_{\rm sim\ rf}$ plus the magnitude of the space charge defocus ${\mathcal{E}'}_{\rm sc}$

$$\frac{{\mathcal{E}'}_{\rm rf}}{{\mathcal{E}'}_{\rm sim\ rf}} = 1 + \xi \quad \text{where} \qquad \xi = \frac{{\mathcal{E}'}_{\rm sc}}{{\mathcal{E}'}_{\rm sim\ rf}}$$

Assuming bunch length small compared with the rf wavelength:

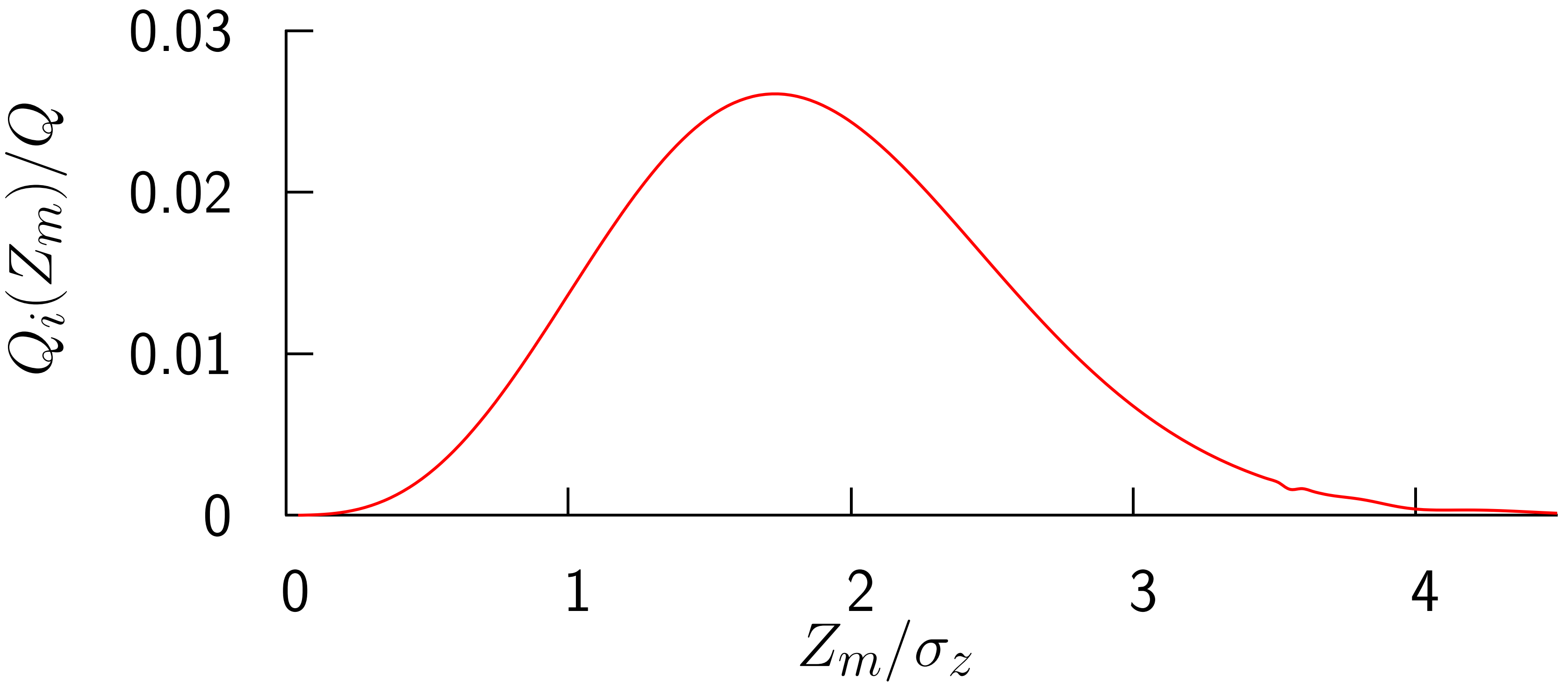
$${\mathcal{E}'}_{\rm sim\ rf} == \frac{\omega\ \mathcal{E}_{rf}\ \eta}{c} \cos(\phi)$$

where ϕ is the rf phase with respect to the zero crossing, ω is the rf frequency, and $\mathcal{E}_{rf}$ is the simulated rf gradient, and η is the fraction of the lattice filled with rf.

$$\xi = \frac{c}{\omega\ \mathcal{E}_{rf}\ \eta \cos(\phi)}\ \mathcal{E}'_{\rm sc}$$

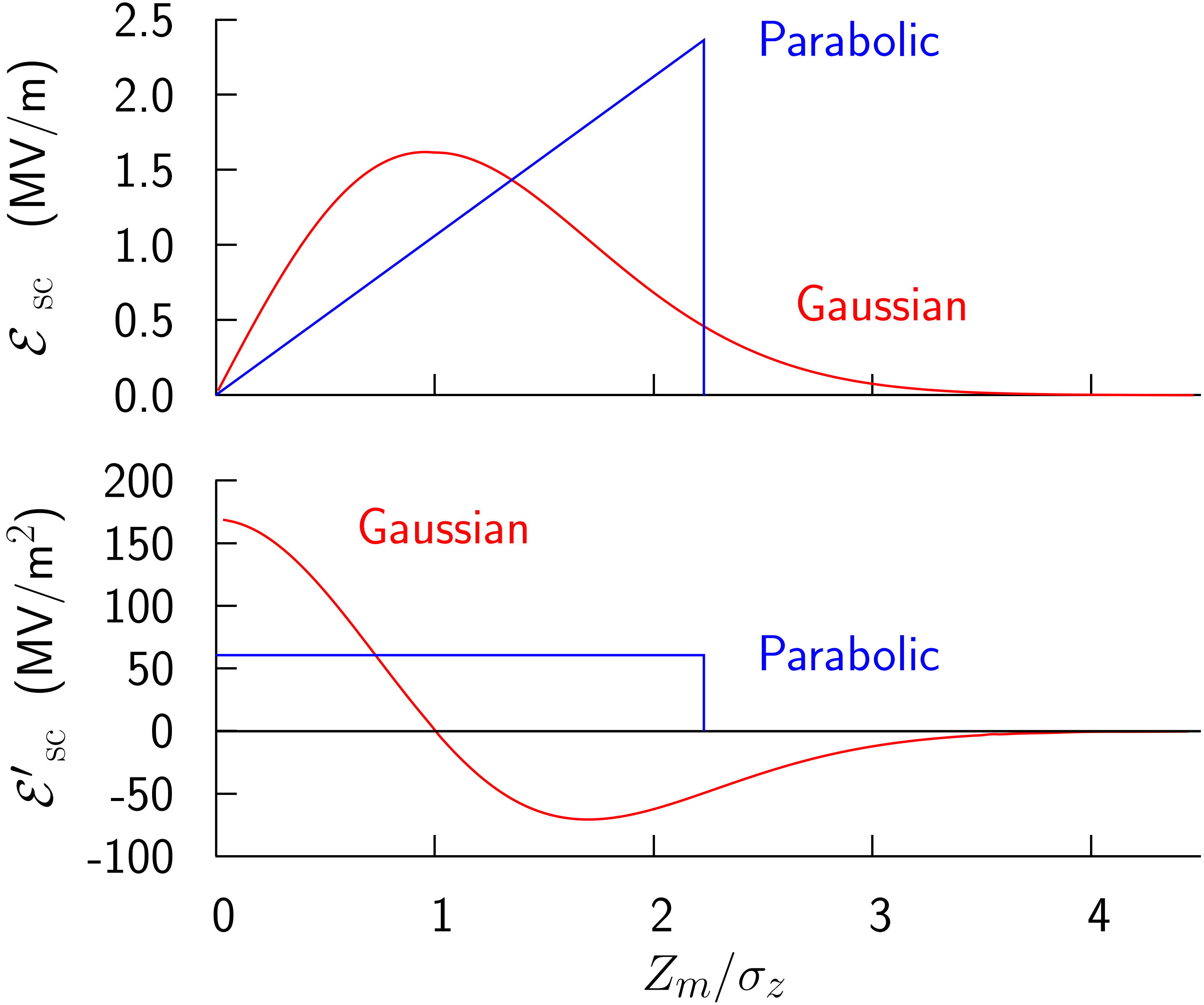
A Gaussian by sum of Parabolics

Since all the above calculations are linear, we can form a sum of parabolic distributions of charges Q_i and lengths z_m that sum to a Gaussian distribution approximating those simulated for cooling in the absence of space charge. For 100 such distributions:

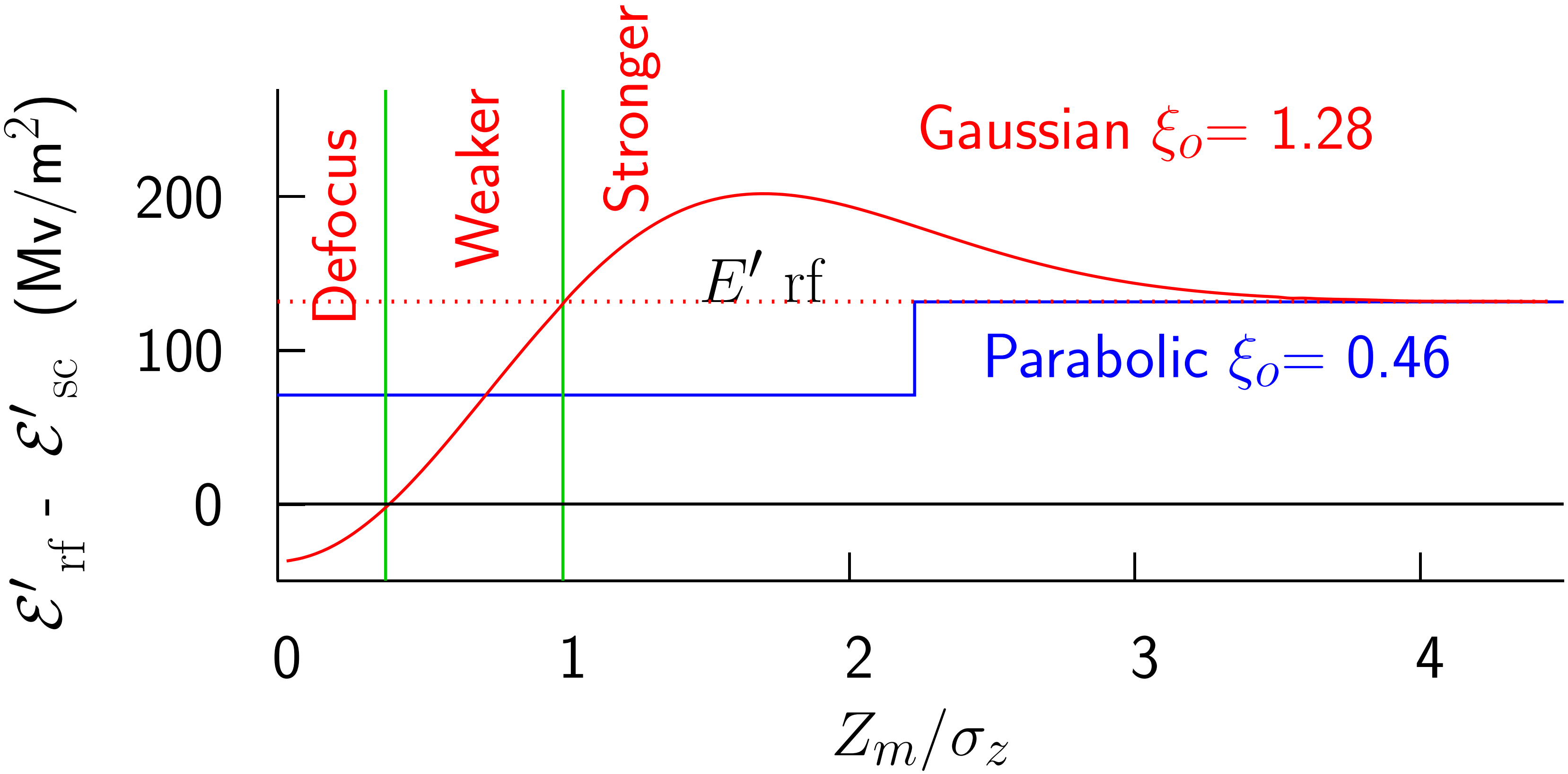


In the following we show calculations for both a Gaussian and Parabolic distributions.

Space Charge $\mathcal{E}$ and $\mathcal{E}'$ for JLAB scheme



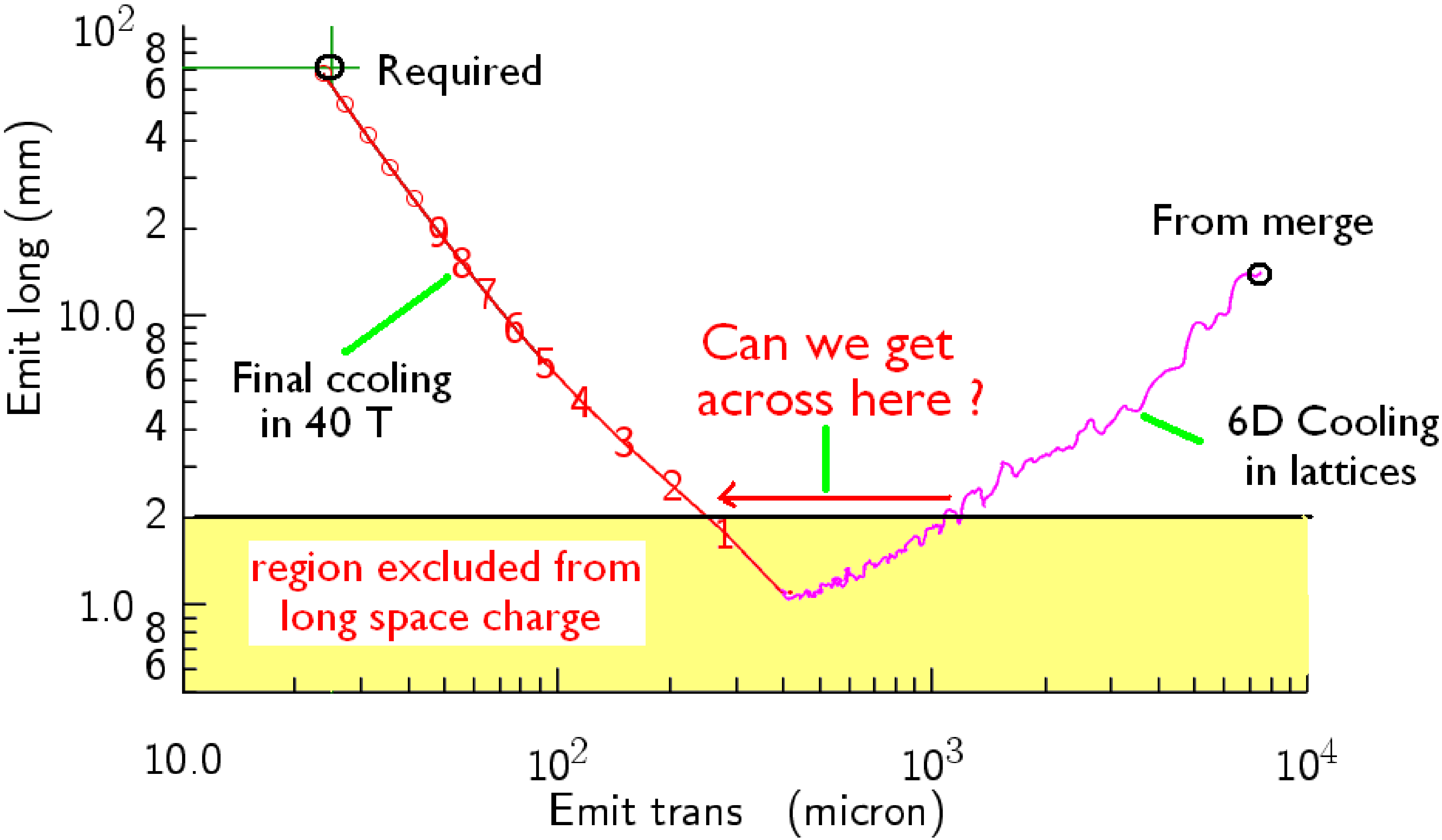
Net focusing with rf for JLAB scheme



- With a Gaussian distribution the bunch center is de-focused !
- But since this condition is reached slowly, the bunch will tend towards the parabolic
- Simulation is required to see if this is acceptable
- But, for the moment we will assume that it is not acceptable

The new cooling challenge

Probably the only way to reduce the Longitudinal space charge is to lower te longtudinal emittance, so the question is

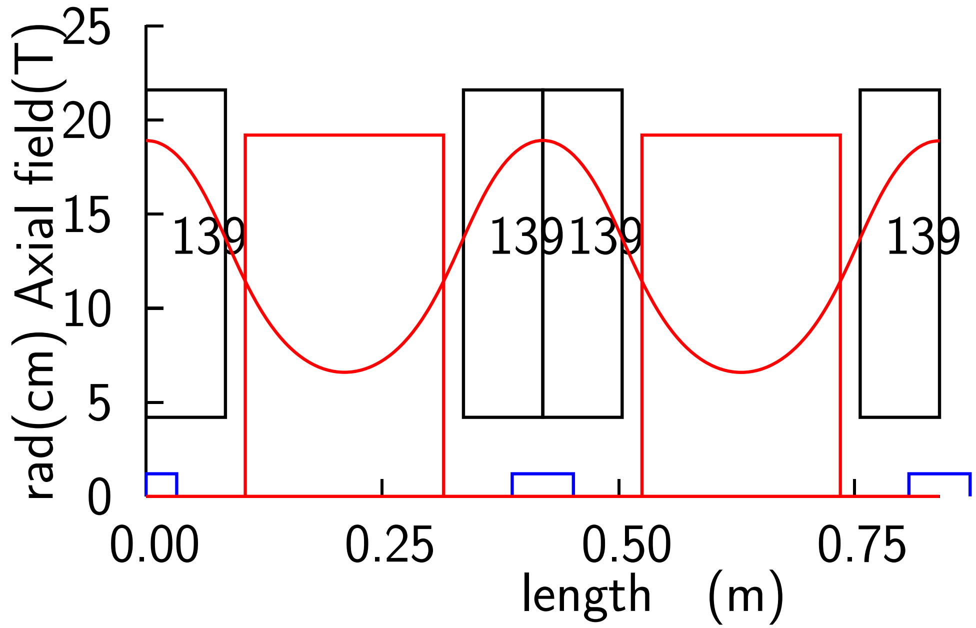


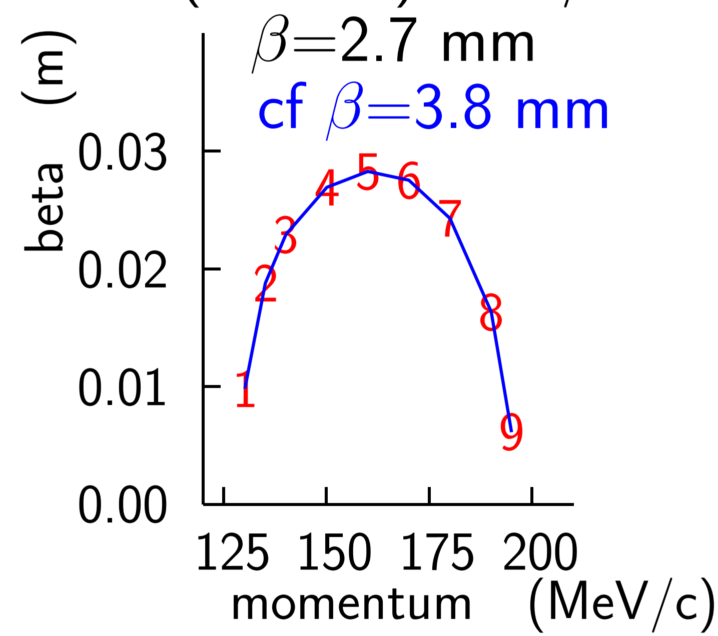
Philosophy

- Keep $\epsilon_{\|} \geq 2 \ \text{mm} \ (\text{vs.} \ 1 \ \text{mm})$ to reduce space charge
- Weaken emittance exchange n tapered cooling after merge, to keep $\epsilon_{\|}$ above 2 mm
- Design special stages for cooling only in transverse direction, but with enough exchange to keep long emit constant
  - Use non-flip (Fernow) cells to lower required fields and current densities
  - Lower momentum to further lower required fields and current densities
  - Reduce cell length, restoring fields and densities, lowering betas, and giving more absorbers per meter

New Non-flip lattice allows 6D to lower $\epsilon_\perp$

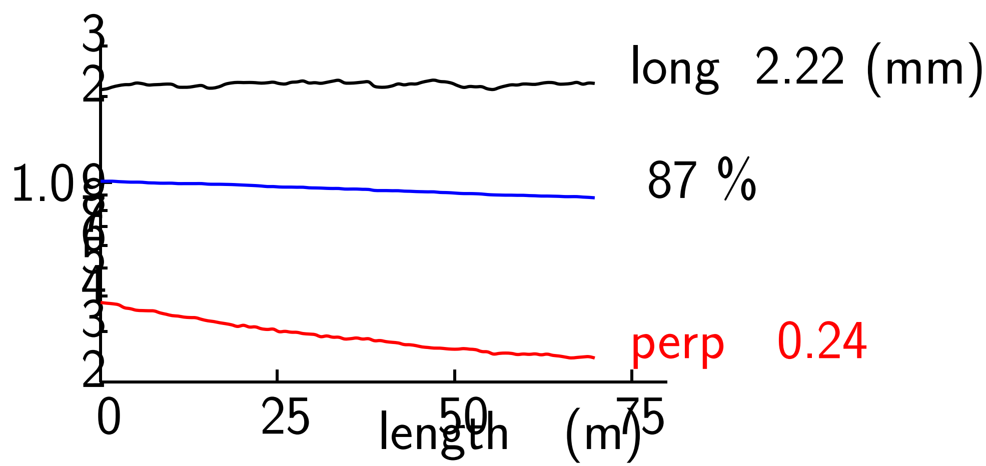
- 42 (vs. 68.75) cm cell, momentum 160 (vs. 200) MeV/c





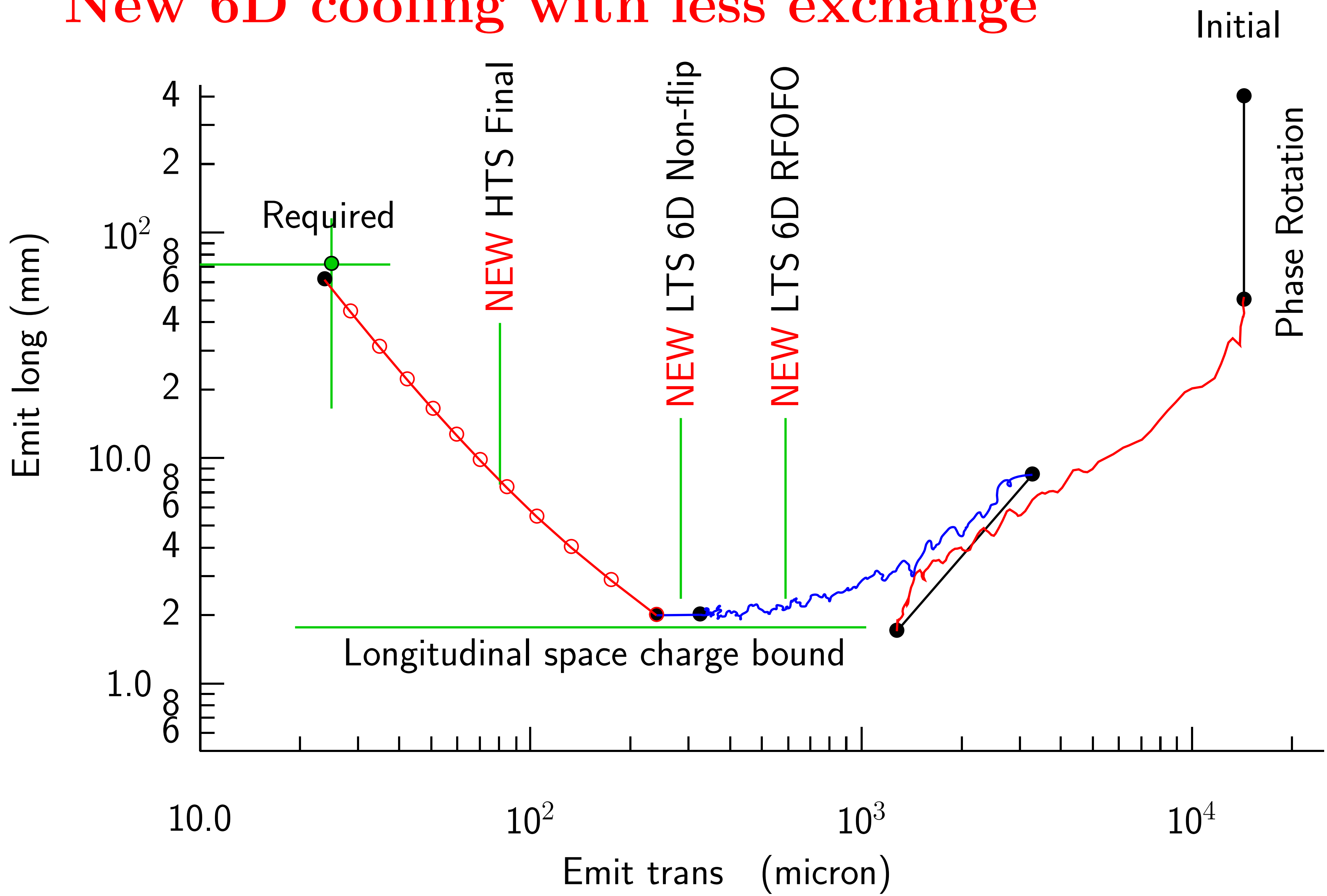
ICOOL

Simulation

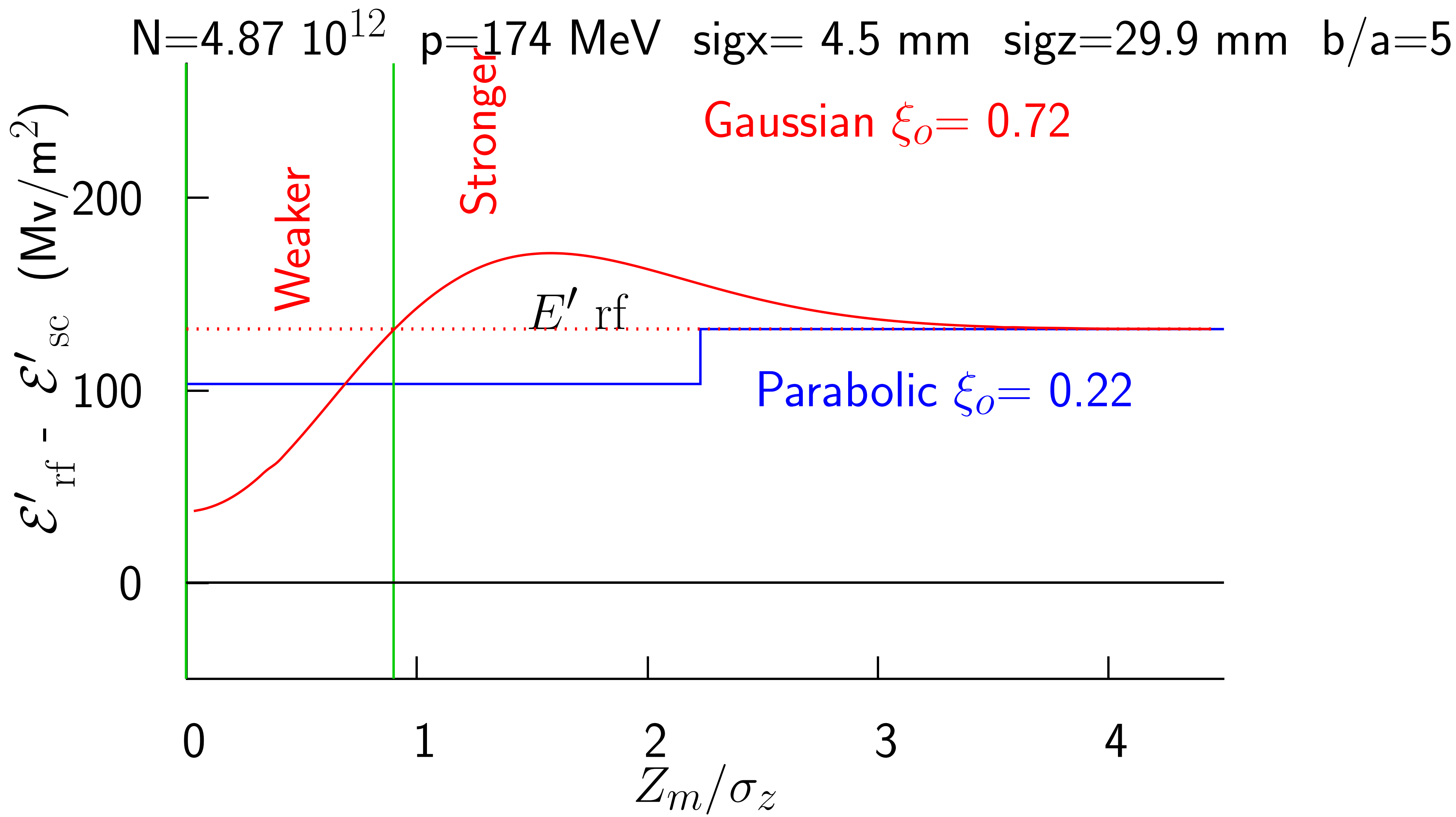


$\boxed{\text{Note Brf=12 T} \quad \text{cf 6 T for RFOFO}}$

New 6D cooling with less exchange

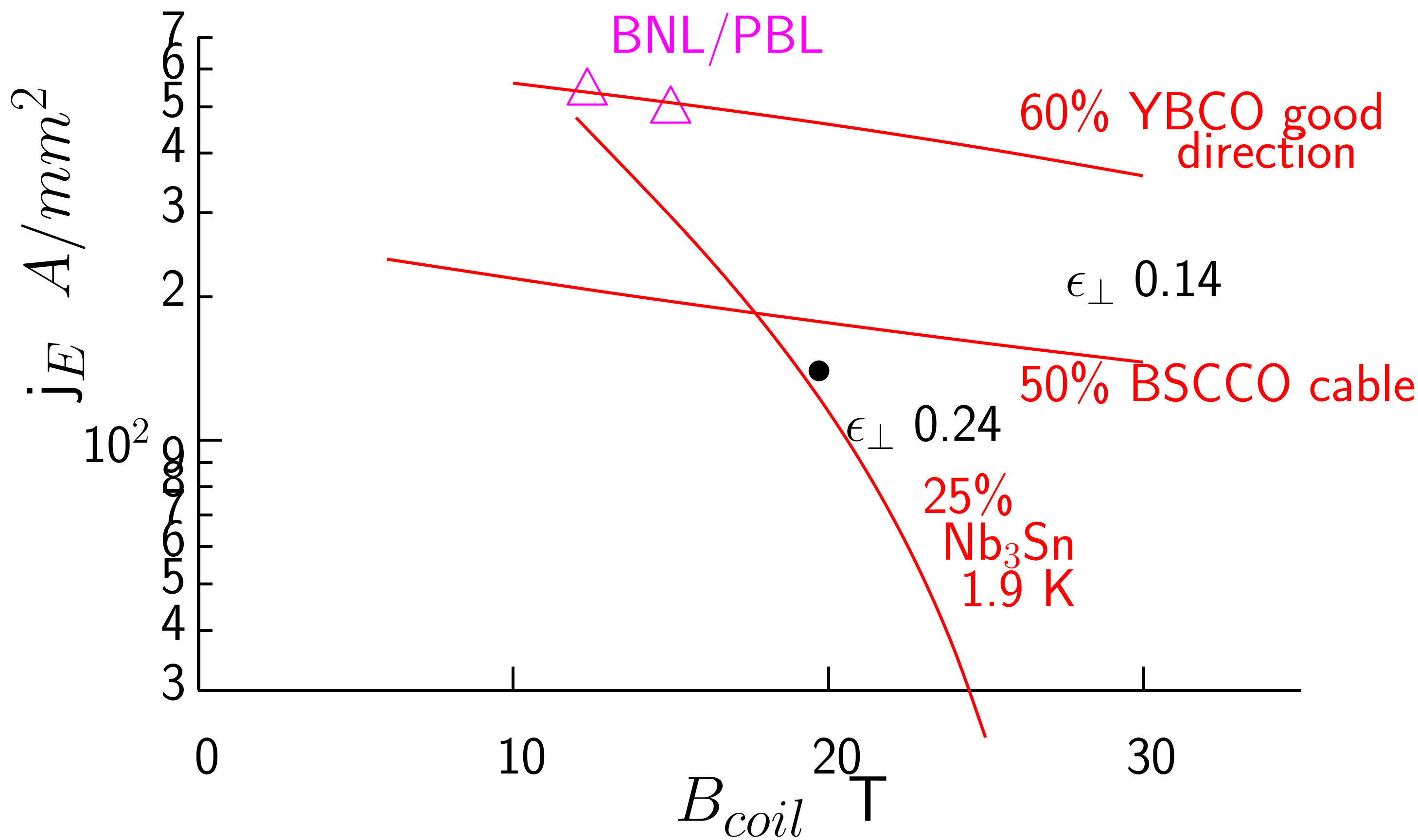


Space Charge with new lattice



- Need rf increase from 20.0 MV/m → 23.9 MV/m
- Assumed acceptable

Superconductor requirements and limits

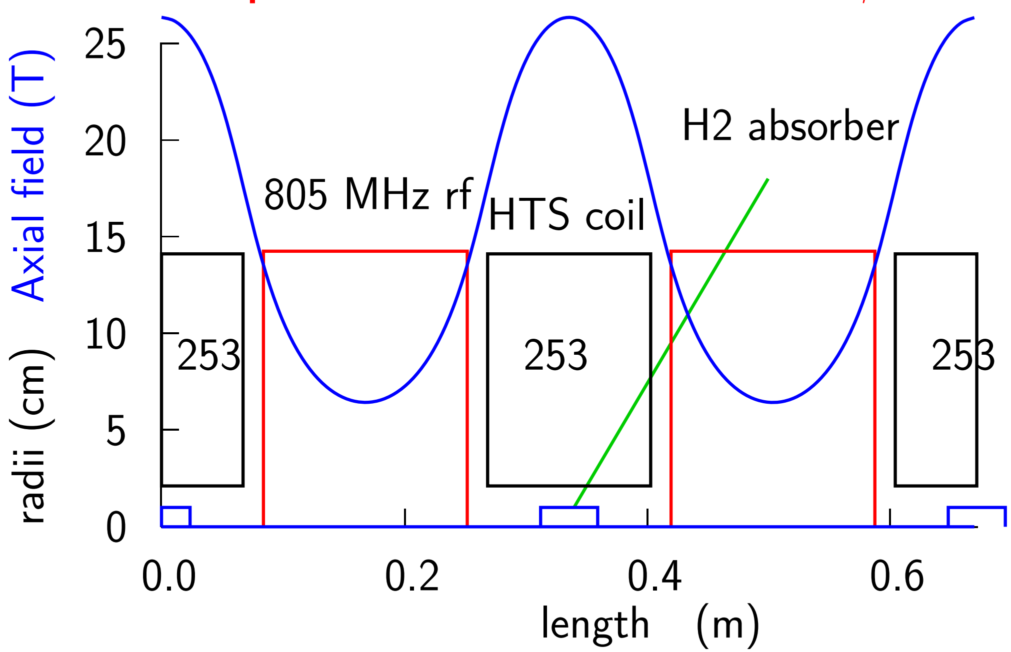


- 60% of YBCO tape justified by BNL/PBL test coils
- 25% of Nb_3Sn for stabilizer & ss from Bob Weggel 15 T design
- 50% of BSCCO assumes ss support, but no additional stabilizer
- Conservative: they assume uniform density based on highest field

Can we do better with HTS coils

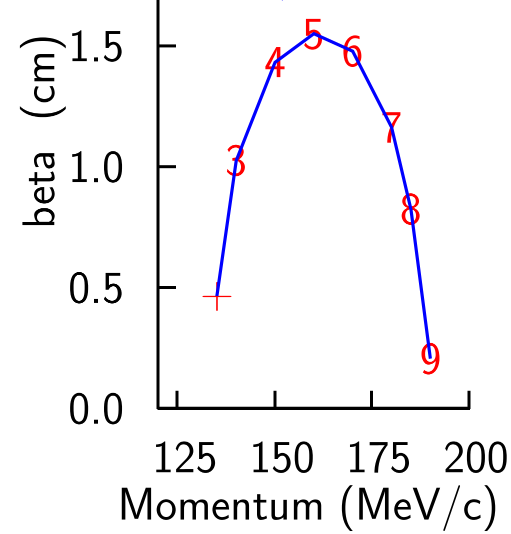
	cell cm	Mom MeV/c	beta cm	emit mm	L cm	r1 cm	r2 cm	j A/mm^2	Bo T	Bmax T
37h	41.0	200	2.8	0.24	16.8	4.2	21.6	174	23.6	24.7
38h	41.0	200	2.3	0.20	16.8	3.8	18.8	197	24.4	25.3
39h	41.0	200	1.9	0.17	16.8	2.6	17.6	199	26.0	26.2
40h	33.6	160	1.5	0.14	13.4	2.1	14.1	253	26.3	26.6

Example of lattice 40h and β vs. Momentum

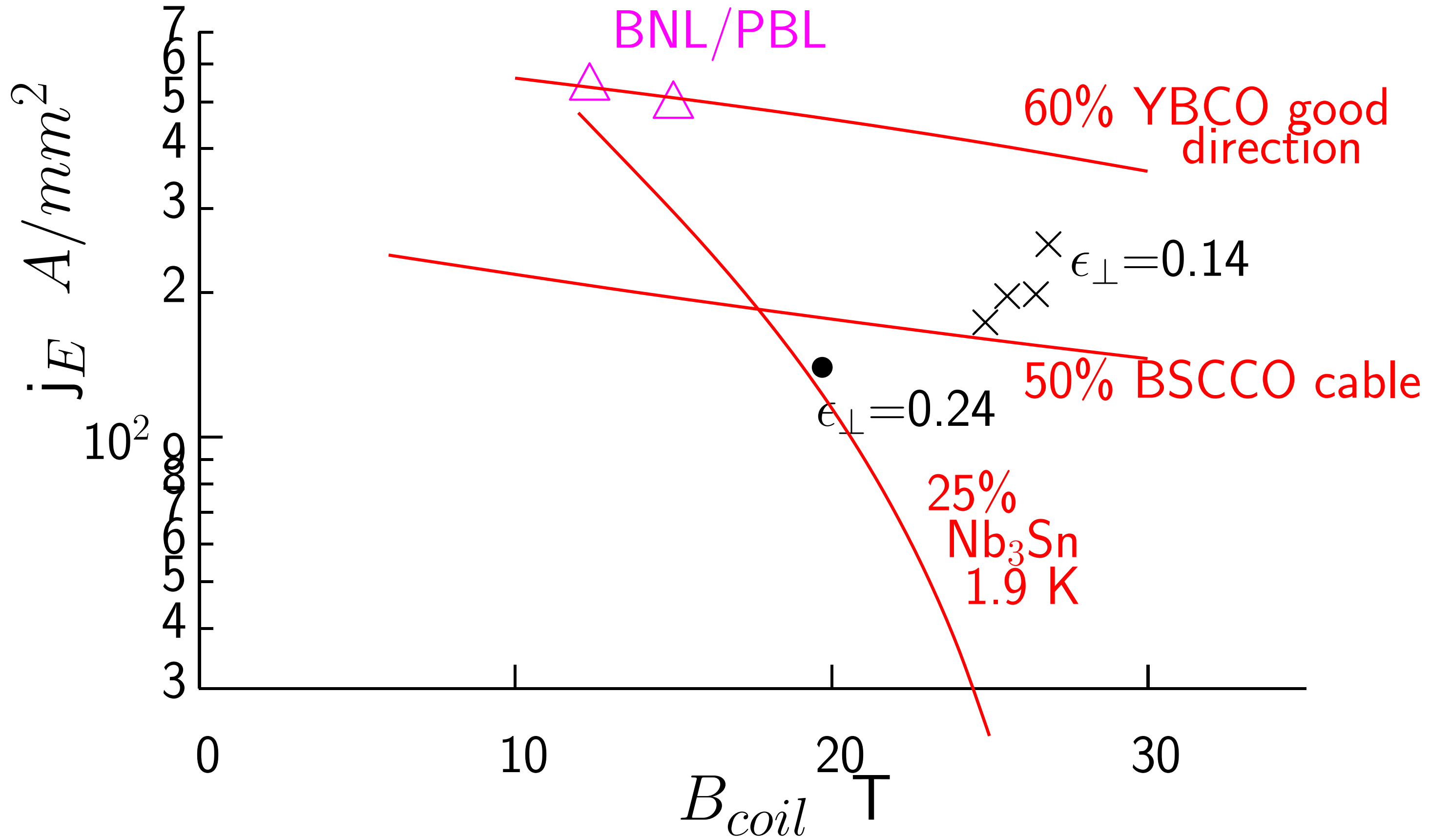


β=1.5 mm

cf β=3.8 mm



6D cooling HTS requirements and limits

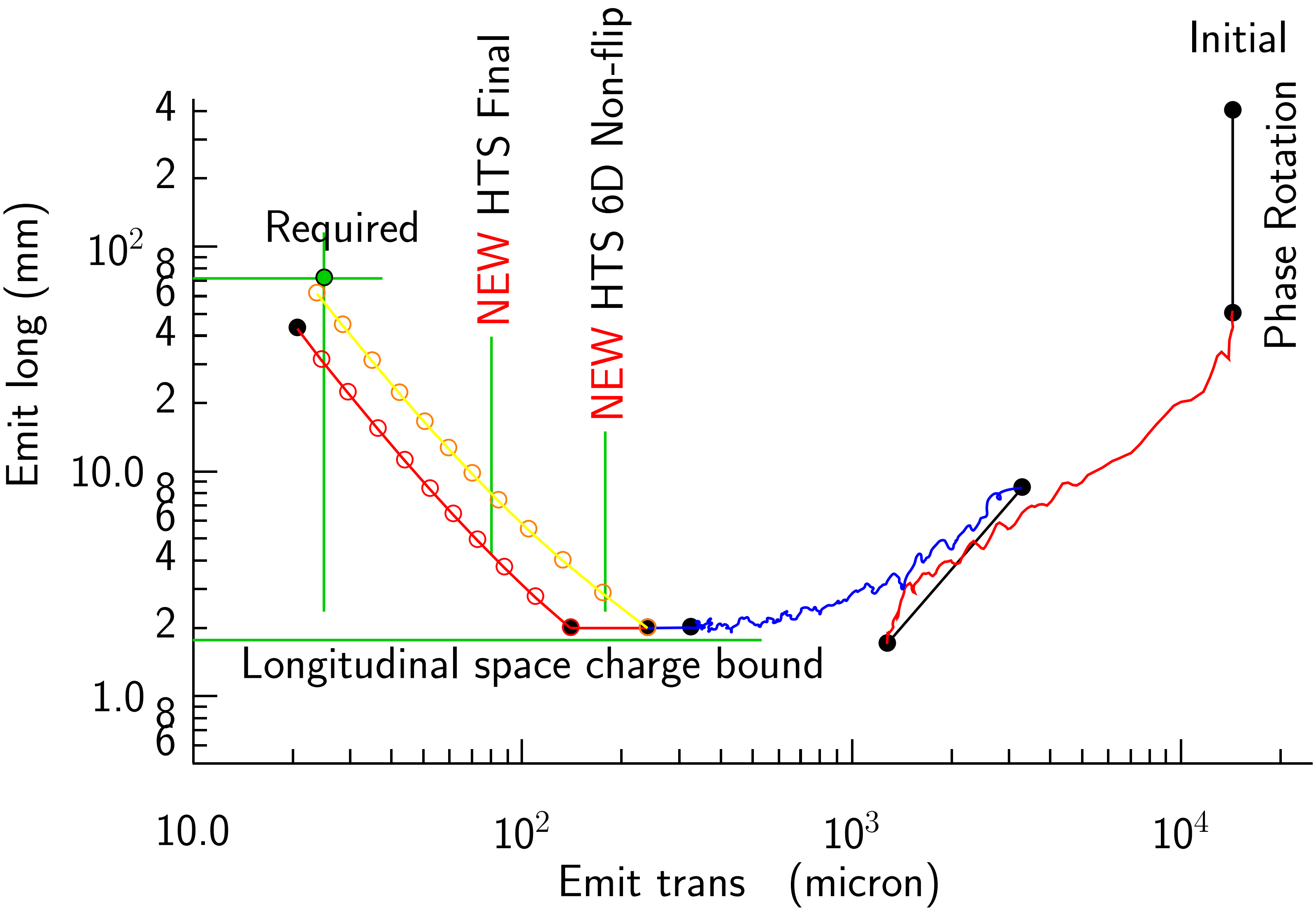


● earlier lattice using Nb_3Sn

× later lattices using HTS

- Only YBCO is suitable for these 6D cooling lattices

With HTS 6D and new Final Cooling



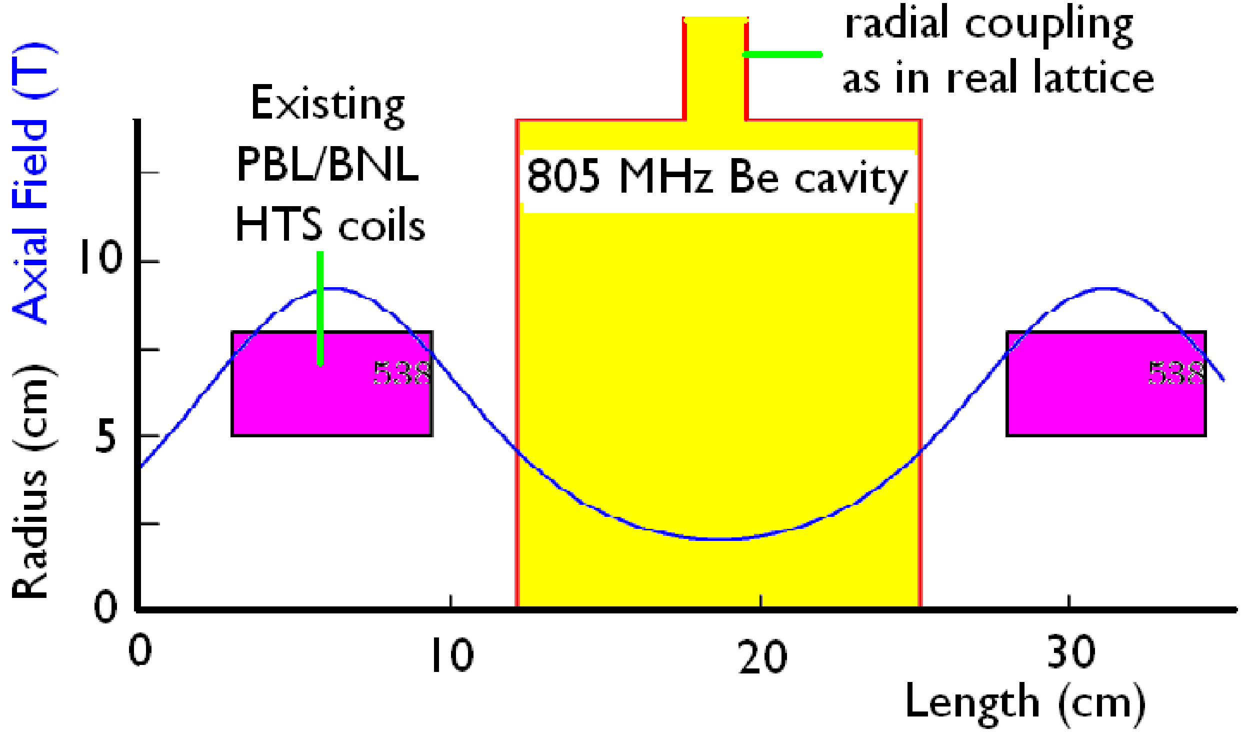
- $\epsilon_\perp$ = 20.7 (cf. 25) μm $\epsilon_\parallel$ = 43 (cf. 72) mm
- Allows more realistic allowances for dilution in acceleration

Conclusion

- Allen et al have full simulations of the parabolic case
- We can express our more Gaussian bunches as sums of parabolic distributions
- Long. space charge for JLAB scheme is probably unacceptable
- A solution is to increase the minimum longitudinal emittance
- We do this by:
  - reducing emittance exchange in tapered lattice,
  - adding a new lattice with non-flip fields
  - but field on rf now ≈12 T (vs. 6 T)
    will discus more on Wednesday
- Longitudinal Space Charge now looks acceptable
- With HTS 6D cooling we can get to lower final emittances giving more realistic allowed dilutions in acceleration

Challenge testing rf in high magnetic fields

- How to get $\approx$ 12 T fields ?
- Use coils developed BBL/BNL coils for 40 T (Gupta)
- First exp uses existing YBCO coils
- Needs side coupled cavity, as discussed by Zenghai Li
- Both require rf at 77 degrees and cryostat



Later experiment

- Also use Nb_3Sn and NiTi coils
- Need Nb_3Sn coils as under design for Phase I outsert of 35 T
- Geometry now near real
- Field on rf 10 T, Field max 18 T, both near real